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# Multi-Objective Optimization of Concrete Shells

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**ABSTRACT:** This paper discusses the application of a Multi Objective Search method to the design of shell structures. Contrasting objectives of light weight and high stiffness are considered for two case studies with different geometry and constraints. The multiobjective search is performed in the frame of the Pareto theory of non domination, implemented in the NSGA-II algorithm. The research focuses on the way a designer can interact with the search computational procedure, driving it towards the desired solution. Diversely from pure optimization techniques, search strategies as Multi Objective search produce sets of feasible solutions, that the designer can know, evaluate and use in his work, accordingly with other architectural requirements and purposes.

## 1 INTRODUCTION

### 1.1 *Computational design of shell structures*

Shell structures, in their different shape and configurations, from thin concrete shells, to glass gridshells with steel or timber grid structure, represent an interesting field for the application of search and optimization methods. During the 20th century, concrete shells have been an important field of experimentation for studying the role of form in architectural and structural design [1]. When only closed form theoretical models and experimentation on small physical models were available, the interest was mainly oriented to structures with regular geometry, as spherical domes, revolution surfaces, cylinders, hypars. Thanks to the developments in computational technique, since last decades of the 19<sup>th</sup> century architects were more and more interested to free form structures in order to widen the field of application and the aesthetic potentialities. The relation between form and structure is nowadays focused on the topics of ‘Non-Standard’, i.e. on new ways of (i) conceiving, (ii) developing and (iii) constructing architectural projects by means of computer technologies. This involves the issue of free-form shapes, which are ‘freely’ designed without any inhibition towards traditional structural and construction principles. Free-form shapes, on the other hand, require computational tools for their representation, as well as for the analysis of their structural or functional performances because the effects of geometrical changes on complex shapes are not easily predictable on the basis of a typological knowledge. The mechanical behavior, as well as other physical performances, related to acoustics, energy, light control, weight, etc. are all strictly depending on the shape of the structure, both at the global and at the local level. The geometric configuration influences the performances as well or much more than the material properties, so that the search of efficient

configurations, in a multi-objective oriented approach, concerns specifically the work of the architect, as responsible of the formal aspects of the design. Search and optimization methods become then a powerful and necessary support for the design process. In previous researches by the authors [2][3][4], different optimization algorithms have been adopted to design complex surfaces, such as folded structures, double curvature surfaces or combinations of both. Optimization algorithms were used considering one performance at a time, the structural response or, separately, the acoustical behavior. In general situations, however, design problems can be interpreted as sets of parallel objectives, frequently in contrast to one another as, to be satisfied at the same time. In structural design typical contrasting objectives are the reduction of weight and the increase of stiffness, or strength. When such situation occurs, traditional optimization algorithms are not suited to find optimal solutions with respect to all objectives. In these cases Multi-Objective Optimization methods [5] must be used.

## 2 BENCHMARKS SETUP

### 2.1 *Geometric parametric model*

In order to understand how MOGA works, the search process is applied to the curved shell structure shown in figure. The shell is a free form surface, defined by a NURBS, which plan projection is a 24m long and 4m wide rectangle. The NURBS is built starting from a set of four spatial curves, two of which are just straight lines, the short sides of the rectangle, while the other two are NURBS curves laying in vertical planes. The four curves act as four vertical sections of the surface to be generated. The surface is defined as a NURBS passing through the section curves (lofted surface), with assigned polynomial degree. Each curve is defined by four interpolating points, which vertical position, the  $z$  coordinate, is variable. By modifying the coordinate of the interpolating point, the section curves change and so the surface. In such a way a set of eight real number is used to completely define the surface shape. The other NURBS parameters of the surface, as the degree of interpolating functions or the number and position of control points, are set constant or directly handled by the algorithm that generates the surface.

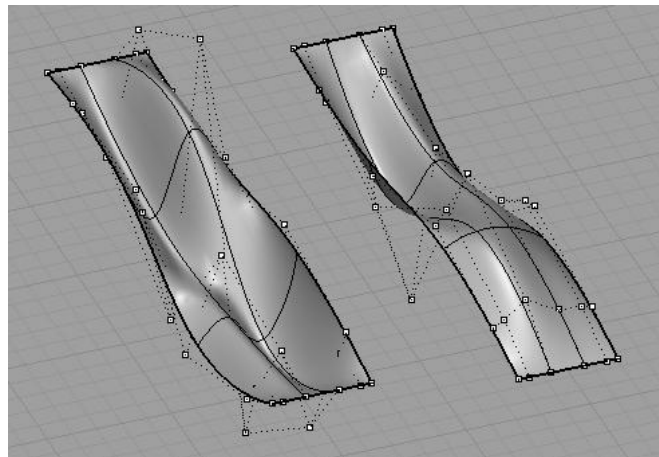


Figure 1. Waved shell used in the optimization process

### 2.2 *Parameters domain*

The ranges of variability of each parameter define the set of potential solution. As it has been already said [5], the width of the solutions domain influences the search process: when the domain is just a narrow layer, round an initial surface, the process is a form-improving, while when it is a large box, the process is rather a form-finding. In the proposed application the search is performed considering a range spanning from -10m to 10m for each variable, corresponding to a true form-finding.

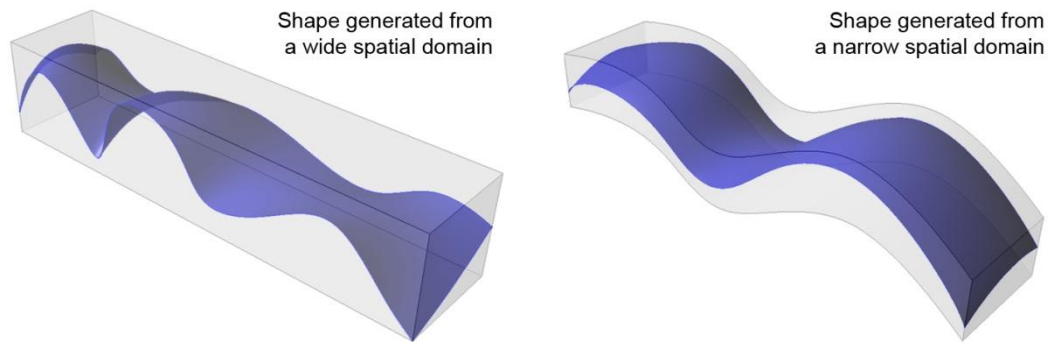


Figure 2: Effect of domain size: form-improving and form-finding

### 2.3 *Finite element model*

The construction of a finite element model finalized to structural optimization presents some differences with respect to models used in normal analyses. The repetition of the analysis for many times is time consuming and, when the problem is relatively complex, it represents a “bottle neck” in the flux of operations. Hence, the first requirement of the model is to be the simplest as possible, with a number of elements strictly necessary to catch the pertinent aspect of structural behavior and with a mesh correctly defined. Even with powerful hardware devices, the repetition for hundreds or thousands analyses can make the optimization problem a hard task, if the model is not efficient. There are basically two possibilities: the use of customized finite elements solvers, developed in the same environment and the use of external applications, as commercial software. Both the alternatives have advantages and disadvantages, but in this part we will consider the first one as the more suitable in order to check the effects of different models on the efficiency of the optimization process. In shell analysis an important issue is the choice of the elements to use: in fact, even for a simple non-layered elastic shell, different formulations and approaches can be adopted.

In the proposed application, the shell is approximated by a net of one dimensional beam elements, which geometric properties are defined in order to reproduce the characteristics of a continuous shell. This allowed to use a custom Python FEM code developed to interact with the parametric modeler.

### 2.4 *Multiobjective optimization*

Multiobjective optimization is based on the concept of Pareto Dominance between solutions.

Shape and characteristics of Pareto fronts can be explained considering a set of benchmarks, in which the fitness landscape is known for every objective and can be expressed by an analytical function.

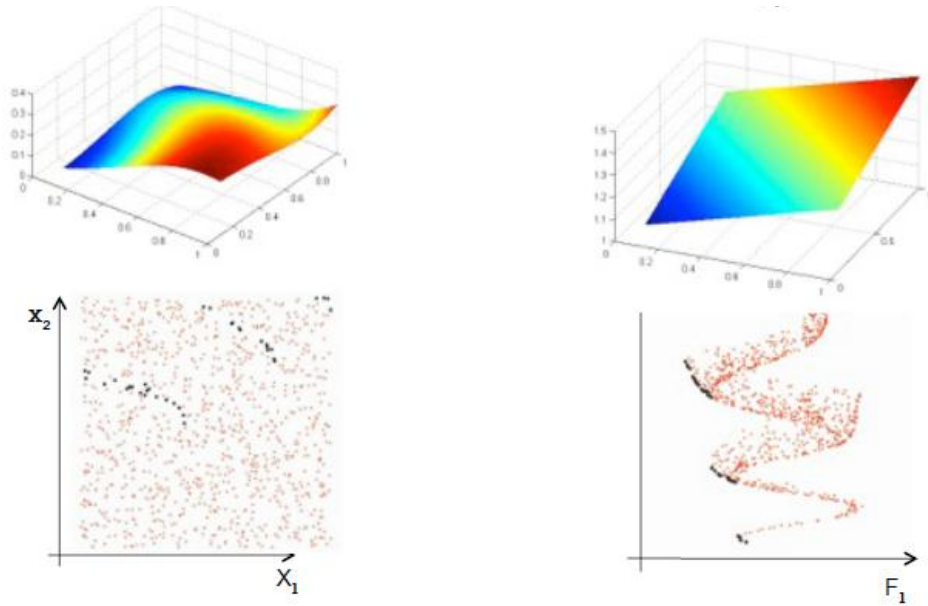


Figure 3: Two variables - two fitness problem: first fitness landscape (top left); second fitness landscape (top right); Pareto front in fitness space (bottom right), Pareto front in variables space (bottom left).

## 2.5 NSGA-II algorithm

Deb, Agrawal, Pratap and Meyarivan, in their article “A fast elitist multi-objective genetic algorithm: NSGA-II”, proposed the Non-dominated Sorting Multi-Objective Algorithm known as NSGA-II. The goal of this algorithm is to generate approximated Pareto fronts of a given multiobjective problem through a genetic optimization process. In order to find an evenly distributed range of solutions in the Pareto front, the algorithm combines the methods traditionally present in a Genetic Algorithm, with the Non-dominated Sorting (NS) algorithm and the Crowding Distance procedure (CD). Considering a finite number of solutions, produced during a given generation step, the Non-dominated Sorting (NS) groups them in a series of sub-groups. The elements of a single group are all non dominated between them, they dominate part of the other groups and are in turn dominated by the rest of the groups. This allows us to organize the sub-groups in a domination relationship, starting with the group who’s elements dominate all of the elements of all other groups, to the group which elements are dominated by the elements of all other groups. Since each sub-group contains elements that do not dominate each other, it can be seen as a Pareto front inside the complete population. The sub-group that occupies the highest position in the non dominated order is the Pareto front of the entire population, and it can be regarded as the current (for the given generation) approximation of the Pareto front of the given problem. Figure xx shows how the NS organizes the solutions.

The CD subroutine guarantees that current solutions are evenly spaced on the Pareto front, avoiding unwanted concentrations. In technical words, can be regarded as a ‘niching’ operator, that preserves diversity of equivalent solutions along all front. After solutions are organized and sorted in sub-groups by NS, CD assigns a further fitness value to each configuration taking into account the distance from the neighbors. The higher is such a distance, the higher is the fitness.

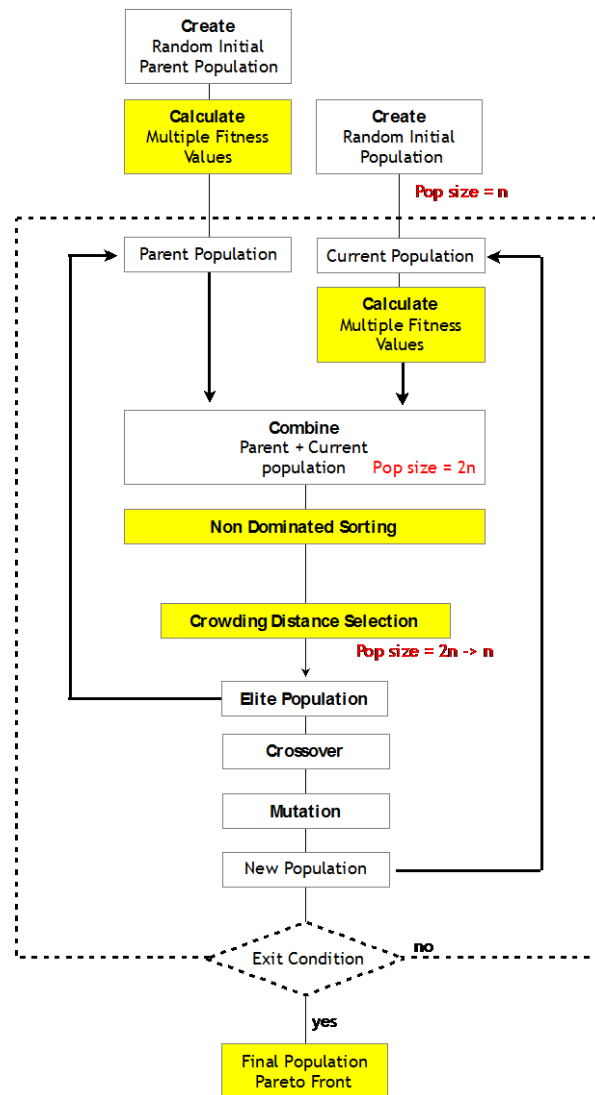


Figure 4. NSGA-II algorithm flowchart

The flow chart in Figure 3 shows the distinguishing characteristics of the NSGA-II. It works with multiple fitness functions, it uses a non dominated sorting algorithm, an explicit diversity preservation operator (Crowding Distance).

### 3 RESULTS

Figure 5 shows the results of the search process for the first benchmark. The Pareto front, represented in the fitness space, contains the best solutions found by the algorithm: the lower branch of the curve contains solution that privilege lightness to stiffness, while in the left branch stiffer but heavier configurations can be found. In this benchmark, the best solution related to each fitness is known: the lightest shape is the flat shape, while the stiffer shape is a dome with the four central point at the top of the domain and the eight lateral points at the bottom. During search process, the CD algorithm tries to keep a good spacing between solution in the front, but the two extremes were not found. The knowledge of such extremes allows to evaluate the efficiency of CD algorithm in terms of ration between the size of the found front and the size of the actual front, including extremes.

The solutions spacing in the front is a good indicator of the variety of geometrical shapes and of the way different shapes answer to multiobjective requirements. A set of such shapes, related to the position on the Pareto front, is shown in figure xx., together with the extreme cases. A direct representation of the position of shapes in the variable space is not suitable, due to the

number of dimensions, but they can be compared one to another by the designer who is in charge to handle the produced material.

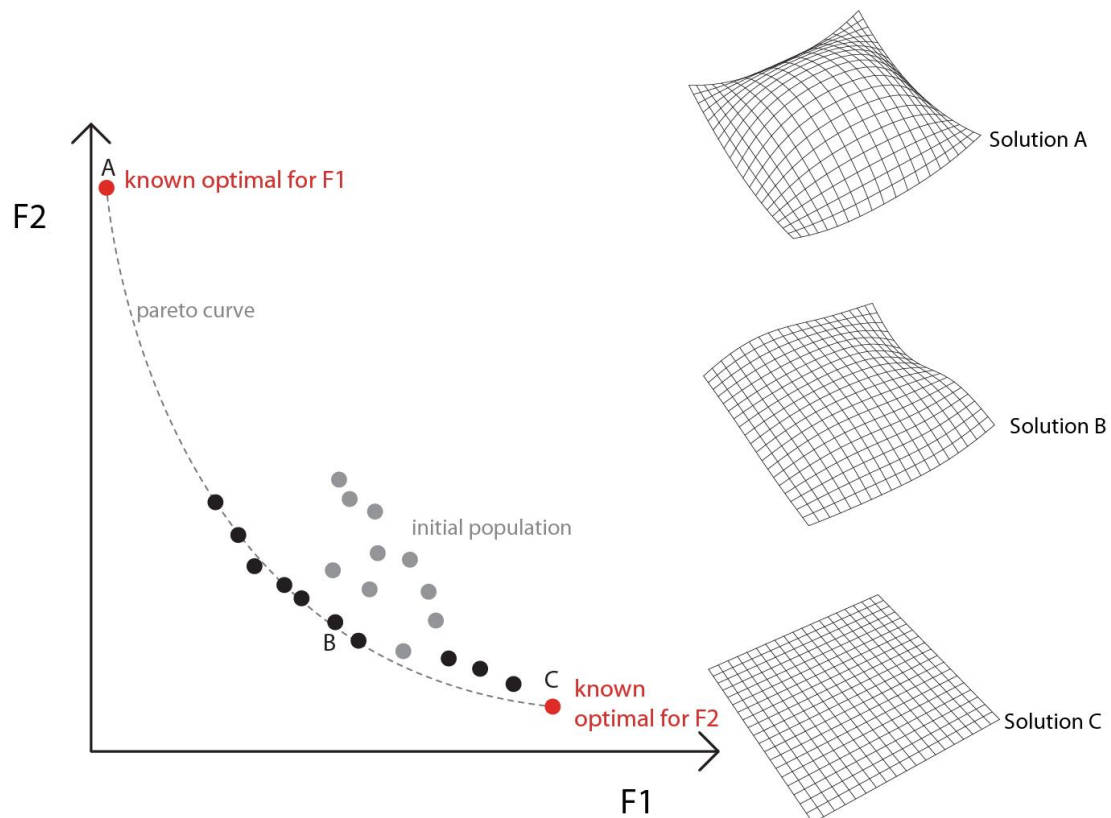


Figure 5. Pareto Front of Benchmark 1

In figure xx the Pareto front, at different stages of the search process, is depicted for the second benchmark, the shell bridge. Even in this case the lightest shape is the flat shape, but the stiffest does not have a theoretical significance. If the longitudinal section of the bridge was an arch with a shape perfectly corresponding to the pressure curve of the load, then the stiffest solution would have this shape and straight transverse section. It would be a barrel vault, or “flat arch”. If the NURBS representation does not allow such a perfect shape, the only way to increase stiffness is to add some bending stiffness, through a transverse waved section. The shell, in this case, becomes a kind of ribbed arch, in which ribs increase the arch stiffness.

Besides the limit case of the stiffest shape, this considerations are important for other Pareto front shapes. Ribs, in fact, increase the stiffness and the weight at the same time.

The set of shapes depicted in figure xx includes some example coming from previous search steps, instead than from the last only: those shapes do not actually represent local minima, but simply steps of the search path. However, they can play a role in interactive design, because they can be chosen as starting points of new search processes, through a redefinition of constraints and of the domain, suggested by the designer evaluations.

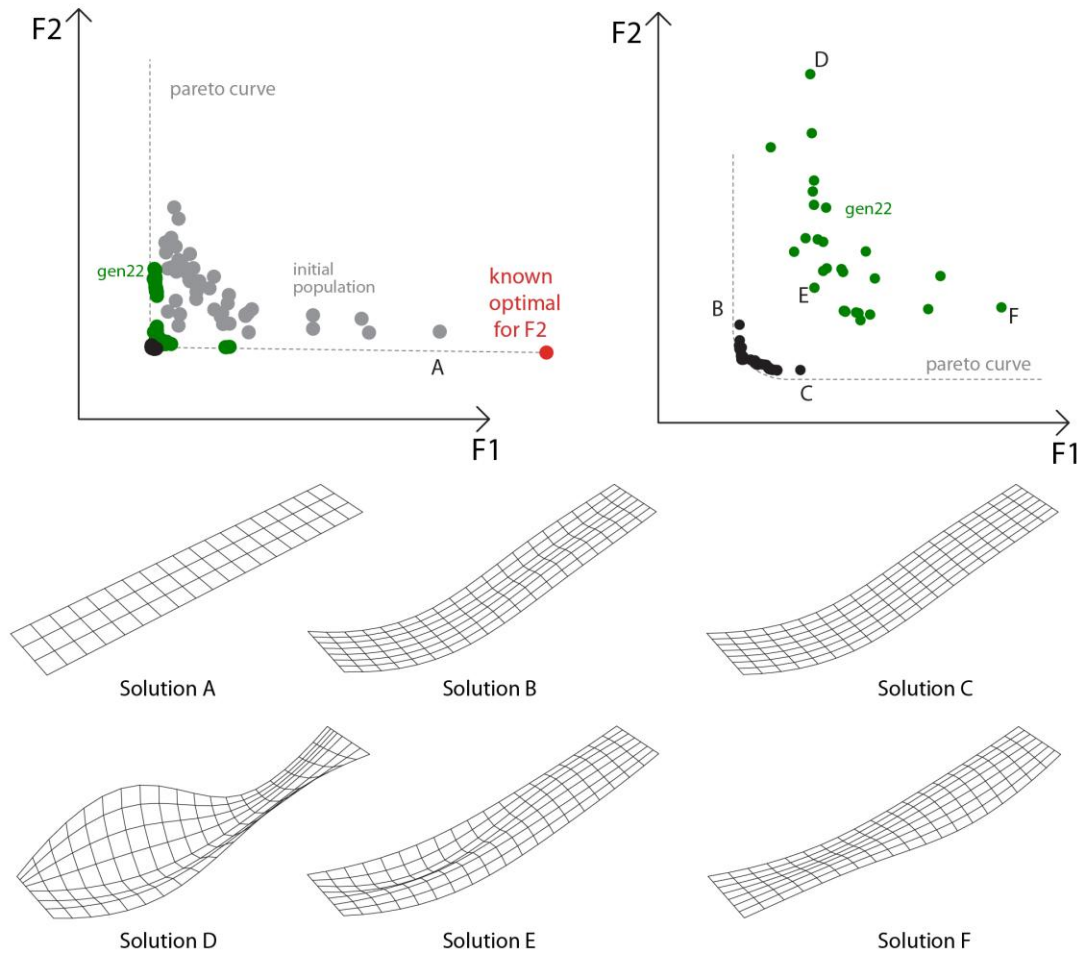


Figure 6. Pareto Front of Benchmark 2

#### 4 CONCLUSIONS

The analysis of the case study shows that the main result of a multi objective search is a set of solutions, characterized by the fact that they cannot be ranked on the basis of the adopted fitness functions. A further decision step is then mandatory to define the final design solution. In the case of performances measures that are in great contrast to each other, the geometrical shapes corresponding to the solutions of the set can be completely different and not necessarily belonging to a uniform family, from the geometric point of view. Hence the multi objective procedure can be regarded as a shrinking of the field of feasible solution, rather than the search of one best solution. The work of the architect as a decision maker, is aided and influenced, but not totally determined by the process.

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